Problem of Points

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Topic: Data and Probability

Grade Level: 1-10

Big Ideas:
Grade 3: The likelihood of possible outcomes can be examined, compared, and interpreted.
Grade 6: Data from the results of an experiment can be used to predict the theoretical probability of an event and to compare and interpret.
Grade 9: Analyzing the validity, reliability, and representation of data enables us to compare and interpret.

Curricular Competencies:

- Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving
- Use tools or technology to explore and create patterns and relationships, and test conjectures
- Use mathematical arguments to support personal choices

Curricular Content:

- Grade 3:
    - Likelihood of simulated events: developing an understanding of chance (e.g., tossing a coin creates a 50-50 chance of landing a head or tail; drawing from a bag, using spinners, and rolling dice all simulate probability events)
- Grade 6:
    - Single-outcome probability events (e.g., spin a spinner, roll a die, toss a coin)
    - Listing all possible outcomes to determine theoretical probability
    - Comparing experimental results with theoretical expectation
- Grade 9: Statistics: analyzing a given set of data (and/or its representation) and identifying potential problems related to bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity

Materials:

- pencils and paper
- 'Problem of Points' sheet (below)
- iPads, open with this coin flipping site open (https://justflipacoin.com/)
- fake money

Problem Description: In brief, this problem involves setting a pair of people up to play a simple coin tossing game. Both players contribute equally to the pot (winnings) and play to an agreed upon number of points. Mid game, before either player has won, the teacher interrupts the students' game and poses to them the problem of points: How do we *now* divide the pot fairly? The problem of points dates back to 1494, and has several solutions to it. The most common solutions include:

1. Dividing the pot according to the probability the players had to win the game, so 50-50.
2. Each player receives an amount of the pot proportional to their wins - independent of the amount of rounds needed to win.
3. Dividing the pot in a way which takes into account the length of the game, and the probability each player had of winning the game.

Students must justify, with mathematical arguments, the way they chose to divide the pot, given that the game was not completed.

Instructions: In groups of two, students are given the sheet (below) called “Problem of Points,” to record the outcomes of their coin tosses. Each player is assigned to win (a point) on either ‘heads’ or ‘tails’, which is determined by the coin flip set up on the iPad. Instruct verbally to the students that the first to three points wins! Once they have started, interrupt their game. If one person wins the first two rounds, we will stop the game then. If they are tied after the first two rounds, we will play one more round before stopping. Then we will pose to them the problem of points: how can we divide the pot in a fair way given the current score?

If you feel that a demonstration of the game is necessary to explain how it works, go ahead! You can use one or two volunteers as necessary to facilitate this.

Variations & Suggestions:

- If students are new to the idea of probability, flipping a coin 10 times and analyzing the outcomes and tracking the data might be enough of a problem. Pose questions like: Do you think you’ll always get these results? If not, what do you think the results would be? What if you flip the coin 10 more times? 100 times? 1 000 000 times?
- Use the graph on this website, https://seeing-theory.brown.edu/basic-probability/index.html, to show a visual representation of actual vs. expected probability. This can help guide students if they are stuck. Be sure to ask open ended questions
- Get students to list all the possible outcomes of a three point game. Does this help them determine how to split the pot? Does it change the way you’d like to split the pot? Can you record these result in any other way? Guide them toward making a tree diagram.
- If students are well versed in probability theory and are enthusiastic, feel free to connect the solution to Pascal’s Triangle. Can they find any connections between their game, their probabilities, and the triangle?
- Ask students to play to a game of 10, and interrupt their game early on leaving many possible outcomes.

Problem of Points

With a partner, we're going to play a coin-tossing game! The first to _____ wins the whole pot. Record your results in the table below.

Your Name: Heads or Tails?	Your Name Heads or Tails?

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

Problem of Points

If you played the whole game:

- Do you think you'll always get these results? If not, what do you think the results of another game would be?
- What happens if you flip the coin 10 more times? 100 times? 1 000 000 times?

If your game was interrupted:

- How do we *now* divide the pot fairly? Is this a fair way to split the pot? Why or why not?