“The Seven Bridges of Königsberg”

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Topic: Geometry/Extracurricular

Curriculum Competencies:
- Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving
- Use logic and patterns to solve puzzles and play games
- Explain and justify mathematical ideas and decisions
- Use mathematical arguments to support personal choices
- Visualize to explore mathematical concepts

Content Objectives:
- Problem Solving (All)
- Concrete or pictorial graphs as a visual tool (Grade K)
- Line symmetry (Grade 4)
- Construction, views, and nets of 3D objects (Grade 8)

Grade Levels: K+

Resources:
Numberphile (2016). *The Seven Bridges of Königsberg* - Numberphile. [online] YouTube. Available at: https://www.youtube.com/watch?v=W18FDEA1jRQ [Accessed 12 Oct. 2019].

TED-Ed. (2016). *How the Königsberg bridge problem changed mathematics - Dan Van der Vieren*. [online] Available at: https://www.youtube.com/watch?v=nZwSo4vfw6c [Accessed 12 Oct. 2019].

West, D. B. (2000). *Introduction to Graph Theory*. Prentice Hall.

Suggested Materials: Cardboard “bridges,” Cardboard “islands,” and Yarn for tracing paths
Alternatively, even just a simple drawing of the Seven Bridges of Königsberg (Figure 1).

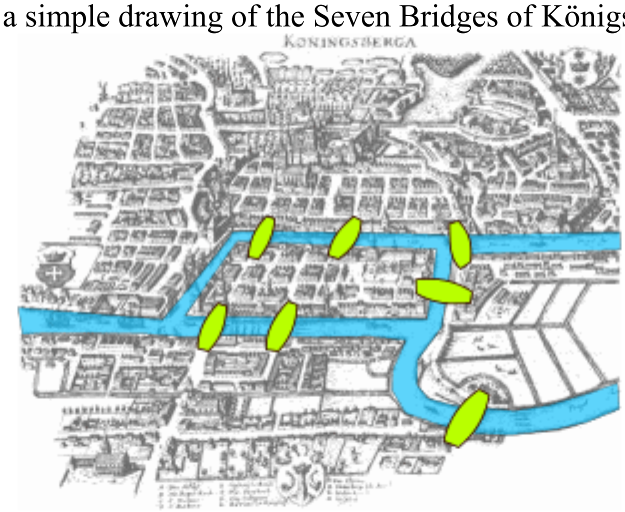

Figure 1: Source: https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

Description:
The city of Königsberg was located on the Pregel river in Prussia. The city occupied two islands along with two areas on both banks. These regions were linked by Seven Bridges (Figure 1). The citizens of the city wondered whether they could leave home, cross every bridge exactly once, and then return home (West, 2000). Is this possible? If so, what is the path someone would take? If not, why is it not possible?

To determine if this is possible, have students walk across bridges and have them trace their paths with yarn. Students should quickly realize that this problem is not easy and is in fact, **impossible**! To help students understand why this is impossible, it may be helpful to present the bridges as connections between 4 different points (Figure 2).

Figure 2: Source: https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

Students should understand that if there were an odd number of bridges connected to a landmass, if they left a landmass, returned, and left again, they would never be able to return. The solution then, is to remove 2 bridges! In fact, during World War 2, two of the seven original bridges did not survive a bombing on the city. This made it possible to walk across all bridges exactly once. Today, five of the bridges remain and only two of them remain from the time of this original problem. Figure 3 shows the 5 remaining bridges in green and the 2 destroyed bridges in red.

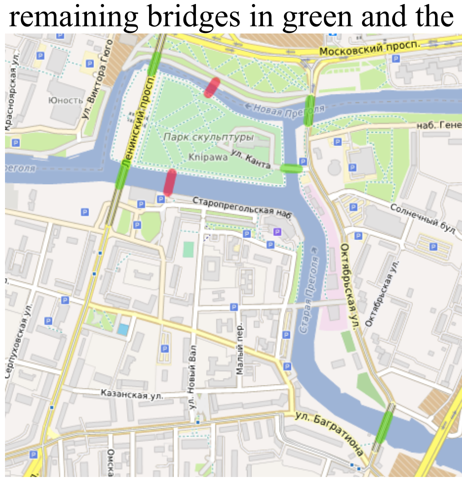

Figure 3: Source: https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

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Extension: Students should then try to find a path with the two bridges removed. If time permits, discuss what would happen if other bridges were removed and note the new start and endpoints of the new layout. Students should see that these start and endpoints are the land masses with an odd number of bridges connected to them.