Hexaflexagons

By Jen Barbour and Jen Whiffen

Level: Elementary Grades

Concepts: Geometry and Measurement, Problem Solving



When making a hexaflexagon, take your time and enjoy the journey. For this exploration you will need a sheet of note paper (8.5×11) cut lengthwise into approximately 2 cm strips, a ruler and a protractor, a glue stick or tape, a pencil and something with which to colour.

Exploration 1: The V-Shape

Let's do some experimenting with those strips of paper!

- Fold 1 strip of paper so that it makes a V-shape.
- Next, hold your V up to a light source. A window or a lamp works well. What sort of shape do you notice? Use a pencil to carefully trace around the shape.
- Measure the sides and the angles. What do you notice? Record your ideas.

There are four possible triangles you might have created: an **isosceles triangle**, an **equilateral triangle**, a **right angle triangle** and a **scalene triangle**.

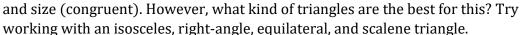
- Which one did you create? How do you know?
- Play around with your other strips of paper. Can you make all four kinds of triangles?
- Is it even possible to make all four? Record your findings.

Use your protractor to measure the angle that sits inside the bottom of your V-shape.

-Can you predict what sort of triangle you made just by measuring this angle?

Exploration 2: Folding Congruent Triangles

In order to make a hexaflexagon, you need to fold a line of triangles that are all the same shape





- -Fold your paper strip so that you end up with a line of congruent triangles. Use a back-and-forth fold instead of a rolling fold.
- -Is it possible to fold all of them? Which ones seem to work best and why?
- -When you unfold your stacks of folded triangles, what do you notice? Pay close attention to how they unfold and what they look like.

If you find the folding problematic, try the template at this website: <u>Click here</u>

Exploration 3: Creating Shapes from Lines of Triangles

You will likely have noticed that two triangles result in a pretty satisfying line of congruent triangles: the right-angle triangle and the equilateral triangle. Play with these lines of triangles to see what kind of shapes we can make!

- -What kinds of shapes result from folding lines of right-angle triangles?
- -What kinds of shapes result from folding lines of equilateral triangles?

Encourage students to test for different polygons and lines! Here are a few examples:



Exploration 4: Making the Hexagon

By now, you have likely figured out that equilateral triangle is the key to making hexagons! To make a starter flexagon, you will need a line of 9 equilateral triangles in a row.

Play around and see if you can make a hexagon. You will notice 1 extra triangle left over. Fold this one over and glue or tape it down. You should have a nice stable hexagon with no major gaps. Now you are going to make sure your hexagon is flexible along all fold lines. Fold it along every line forward and back to make sure it is good and flexible.

You should be ready to pinch your hexagon so that you make a three-pointed star. Gently open the top of the 'star'. Open it all the way. You will see...another hexagon! Pinch it to make another tree shape and repeat.

Exploration 5: Playing with Designs

Grab some pencils or markers and start to colour your hexagon. Colour both sides a different colour. Add some extra designs if you like.

- -Pinch and open your hexagon now. What do you notice?
- -When you have coloured all possible surfaces of your hexagon, you can even unfold it to see what happens. Before you do it, make a prediction: what do you think it will look like?
- -Make more hexaflexagons! Create different kinds of designs! Play and experiment. What do you notice?

Finally:

Challenge yourself! Can you make a double hexaflexagon that shows six possible designs instead of three? What about a triple? Is it even possible? What happens when you try?

Task adapted from and inspired by:

How to fold a hexaflexagon. (2020, Sept. 6). In *Wikihow*). https://www.wikihow.com/Fold-a-Hexaflexagon

Vihart. (2017). *How to make a hexaflexagon: A definitive guide.* [Video].

YouTube. https://www.youtube.com/watch?v=Svg2Kscmmwc